may be even more potent than it is reckoned. With associations growing in influence, and the great facilities afforded for exchange of ideas, the body of teachers is very rapidly increasing in strength, and this reform in the teaching of mathematics, together with many another much-needed reform, is perhaps much more in the immediate future than is thought. At any rate, if the bow and arrow is still the official weapon, the use of the magazine rifle is being secretly taught, and we school teachers look forward with no misgivings to that great fight. Prof. Perry sees ahead for our people, rather we are "spoiling for it," for with it will come our freedom!

Frank L. Ward.

1 Macdonald Place, Hartlepool, March 29.

Rearrangement of Euclid Book I., Pt. i.

In answer to Prof. Lodge's letter I should like to say that we have for some time followed much the order he suggests. Euclid's order unnaturally separates propositions which should come together, *e.g.* I. 4, 8, 26, and is, therefore, a serious hindrance to a clear grasp of the subject-matter as distinct from mere exercise in logic.

The following order—substantially that suggested by Prof. Lodge—seems natural, and we have certainly found it work very well in practice.

(1) The propositions on angles, viz. 13, 14, 15, 27, 29, 32, cor. 2, 32. At this stage logical deduction from definitions and axioms is difficult and, to a boy, unconvincing. The following proof of I. 32 cor. 2 is convincing, at least: "If a man walks right round a rectilineal figure (starting and ending at a point in the middle of a side), he turns once round. Hence the exterior angles, which are the angles through which he turns, are together equal to 4 right angles." Similar proofs of 27 and 29 are equally convincing. Any attempt to analyse these proofs into the axioms on which they depend seems to me at this stage foolish; it is work for a highly trained and speculative mind, not for a boy.

(2) Triangulation, I. 4, 8, 26.

These are, I think, best presented as the outcome of experience passing into intuition, and as special cases of the general fact that three data are necessary and sometimes sufficient to determine a triangle. The special case of right-angled triangles with hypotenuse and one side given should be added and proved deductively from I. 5.

The rest of Book I. consists of exercises on these fundamental propositions:—Properties of a single triangle, I. 20, 5, 18, 6, 19; loci; quadrilaterals; areas. The order in which these last three subjects are taken is immaterial.

A special advantage of this arrangement is that it makes it easy to combine practical with theoretical work. It was, indeed, from the attempt to do this that we were led to follow this order, but even in purely theoretical work it has proved a great gain.

As to the omission of "constructions" from the deductive course, we agree—they are properly treated as exercises.

As to the effect of this change on real progress we have no doubt. As to examinations, we hope that they will before long (1) permit freedom in the order of propositions, (2) diminish bookwork and insist upon riders and practical work, as some, indeed, already do.

It seems illogical, but even in deserting Euclid's order we adhere to his numbers. The constant reference to cardinal propositions is a great help to thoroughness and clearness of knowledge, as well as to ease of questioning and answering. Probably no one will ever succeed in fixing fresh labels on to the propositions, and for the present at least we find the old ones useful, though they are to our boys quite arbitrary.

W. C. Fletcher.

Liverpool Institute.

I quite agree with Prof. Alfred Lodge as to the order of propositions he proposes, which is practically the order I adopted in my "Foundations of Geometry." But he does not in his letter refer to what seems to me the chief reason for it, which is that the elementary geometry of straight lines and angles should precede the geometry of plane surfaces, including any propositions about areas. And to carry out this idea, the fundamental propositions which Euclid gives so badly in his XIth. book (props. 1–9) ought to be taken before such propositions as his I. 35 and 36. On the other hand, there are important propositions in the XIth. book, notably prop. 10 (if this is not included in the definition of parallelism) and props. 20 and 21, which come properly in what Prof. Lodge calls the first part of Book I.

By the way, I may mention that it seems to me illogical to prove I. 27, as Prof. Lodge does, by a simple "which is impossible," and to refer I. 29 to "Playfair's axiom." Neither proposition is nearer *a priori* truth than the other, and it is just as easy to disguise the difficulty, if you wish to do so, in either case.

Edward T. Dixon.

Racketts, Hythe, Hants, April 14.

We have had the following arrangement of Euc., 1–32, in use for three years with more than two hundred pupils. 13, 14 (from the definitions); 15; 32, cor. 2, 32, 16, 17; 23, 8, 9; 4, 10. Locus of a point equidistant from two given points. 11, 12, 5; 26, 6. Locus of points equidistant from two intersecting straight lines.

This gives fourteen propositions; thirty-seven more complete all the plane geometry of Euc. I.–VI. and XII. required in mathematics or science. We have no superposition "proofs"; they merely obscure obvious truths. Parallels by superposition have been found beyond the capabilities of beginners. Why not alter the definition? At present it gives the least obvious property of parallels.

A caution to the professors who are teaching us how to teach. We are seeking a system of geometry suitable for boys of ten, and the most logical method is not necessarily the best; it is better to separate 4, 8, 26 by examples of their use and to leave the remaining case for trigonometry. Again, an ideal course must be inventional, and must grow out of practical work; therefore it must introduce problems as early as possible: a beginner should not be allowed to quote a construction which he cannot perform. Is not the demand for a purely theoretical course due to a desire to use 1, 9, in proving 1, 5, whilst retaining Euclid's proof of 1, 8?

T. Petch.

Leyton Technical Institute, April 14.

In reply to the appeal of Prof. Alfred Lodge for opinions with reference to his proposal to alter the sequence of Euclid's propositions by introducing those relating to parallels at the earliest possible stage, permit me to express what I hold to be insuperable objections to his proposed innovation.

Whatever other objections may be raised to Euclid's sequence of propositions, it at any rate has this distinguishing merit, that it separates the propositions (I. 1–28) which are independent of the postulate of parallels from those which are true only when that postulate is admitted. To obscure this distinction, as, for instance, by treating props. 16, 17 as corollaries of prop. 32 and so appearing to depend on the postulate of parallels, would to my mind, especially now that the non-Euclidean geometry of Lobatchewsky and others is an established part of mathematical science, be a distinctly retrograde step.

Further, this innovation is not in the least necessary to secure Prof. Lodge's object (with which I entirely sympathise), namely, a better and more natural grouping of the propositions about triangles.

For this purpose all that is necessary is to add I. 16 to the three (13, 14, 15) with which he proposes to begin. This proposition may at once be proved as follows:—

The triangle being A B C, the side B C produced to D and E the mid-point of A C, turn the triangle A E B about E until E A comes on E C and A on C, then E B comes to a position E F in the same straight line as B E, and since B E F, B C D meet in B. they cannot meet again, so that F lies on the same side of B D as A [N.B., here comes in the difference between plane and spherical surface geometry], and E C F or the angle A is less than the exterior angle A C D.

This proved and I. 17 as its corollary, the propositions about a single triangle and those about the comparison of triangles easily fall into a simple and natural sequence and grouping.

Shanklin, April 12.

Robt. B. Hayward.

Winter Phenomena in Lakeland.

There being no record within my knowledge as to whether holly and ivy are starch-trees or fat-trees, *i.e.* as to whether their wood-starch disappears or otherwise in winter, a strict watch was set upon the phenomena. During the months of December, January and February, sections were taken at

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intervals of the twigs and smaller branches of these trees, and their content in starch was carefully tested and observed. The general result was that the ivy is a decided starch-tree, inasmuch as at no time during the winter was its wood found free of starch or anyway nearly so. No doubt, as compared with summer, there was a great diminution of this substance, many of the medullary ray cells being completely empty of granules, and moreover, *e.g.* on January 18, only about one-quarter of these cells were stored with starch, and even this gave a reddish-brown tinge to iodine, as if amylodextrin was intermixed therewith. As regards the holly, there could be no question that the amylaceous reservoirs of its wood had suffered a still more serious depletion. Nevertheless, they were never found completely void, a more or less considerable accumulation of starch granules being readily distinguishable along some portion of the medullary rays; the pith, too, was never empty (that of the ivy was found invariably depleted). It may be added that in both cases starch was completely absent from the bark during the winter months.

The overthrow by the boisterous gales of the departing year of a crab-apple that was perched aslant on a sharp ridge of rock afforded an opportunity for observation of the root appanage of this tree. The wood of the root contained a considerable amount of starch, but none was found in its bark. The alcoholic and aqueous extracts of the latter proved very rich in phloridzin, the reactions thereof being yielded with eminent distinctness and beauty, and the phloretin prepared from it indicated by its behaviour in alkaline solution with nitroprusside of sodium that it was a fatty aromatic ketone. Phloroglucin was detected in the wood, but none in the bark.

Whilst exercising on the hills on January 19, numerous faded and embrowned brackens were seen which a week or so previously had been overlaid with a thick mantle of snow. The fronds of one of these were incinerated, and the crude ash was found to contain no less than 43·6 per cent. of silica, also 13·2 CaO and 0·8 phosphorus (or 1·8 P_2O_5). This enormous amount of silica is all the more remarkable, inasmuch as in August the ash of the fronds contains only about 9 per cent. of this constituent and that of the stem about 7 per cent.—the difference is doubtless to be attributed to the very considerable diminution of the soluble salts, and not of the lime, during the course of autumn. In this case, as in that of all calcifugous species, a strong proportion of lime in the young and vigorous frond would offer a serious obstacle to the presence of a considerable quantity of potass. Thus, on June 3 the amount of potass and soda in the ash is somewhere about 32 per cent., in August 27 per cent., and in late autumn about 2 or 3 per cent. The lime, on the other hand, fluctuates from about 6 to about 13 per cent. only, and thus while in a great many leaves the autumnal deficit is, so to speak, covered by lime, here in the case of the bracken it is refunded by a lavish plethora of silica. The general inference is that the frond, having been utilised as a support for the reproductive parts (sporangia), the strain thereby involved leads to a very advanced condition of decay, whence a tremendous drainage of silica towards the organ. Perhaps it is this very faculty of remaining decayed without disfigurement, encased in silica, that is one of the causes why abundant remains of ferns are found in the fossil state.

Patterdale, Westmorland. P. Q. Keegan.

The Species Problem in Corals.

May I direct the attention of your readers to an exhaustive article, dealing specially with "species" among the reef-corals, by Prof. Döderlein in the current number of the *Zeitschrift für Morphologie und Anthropologie?* It is somewhat hard to find myself there blamed for continuing to use the word "species" in relation to a group in which it is totally inapplicable, for I laid the whole subject before the Linnean Society a year ago. I stated the difficulty and suggested a provisional alternative method of designating specimens. I carefully explained that I was proposing a method of work in order that we might discover the true species by gradual arrangement of the variations. In the autumn of the same year I made a brief statement at the International Congress of Zoologists in Berlin, but the report has not yet appeared; and again, later in the year, I read a paper before the Cambridge Philosophical Society. This has appeared in the last number of the *Proceedings* of that Society under the title "On the Unit of Classification for Systematic Biology."

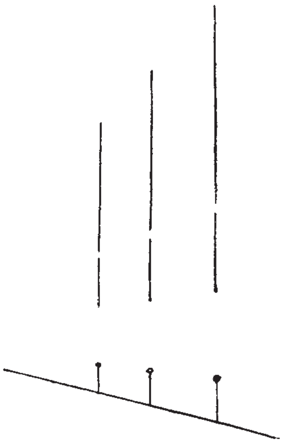
It will be noticed that while Prof. Döderlein discusses the general problem very fully, he makes no practical suggestion as to how to designate the different forms for reference. I, struggling with nearly a thousand foolscap pages (dealing with three genera) for the "Catalogue" of the Great National Collection of Corals at South Kensington, was compelled to invent some way of labelling them. But before finally adopting it, I sought the advice of many of the leading zoologists and botanists within reach. Henry M. Bernard.

Clapham, S.W., March 21.

Sun Pillars.

On January 19, 1901, during zero weather at Winnipeg, Manitoba, the fine icy particles which floated over the city as a cloud at 11 a.m. gave a halo round the sun with two mock suns to the right or left at the same altitude as the sun, and above the halo an inverted arc of about 120° showing prismatic colours. There was a good deal of wind at the time.

At night the wind dropped and the air became very still. The haze of icy particles descended, and in the middle of the night its illumination by the strong arc lamps of the city gave a column of light, comparable to a sun pillar, extending above each lamp to a height estimated at 300 feet. A striking feature of the phenomenon was the perspective effect, the columns diminishing in apparent height in such precise proportion to the increasing distances of the street lamps that it was evident the upper surface of the cloud of particles was truly horizontal. At this time the vertical columns of light were also visible below the lamps and extended to within a few feet of the ground. A few hours later, viz. at 4 a.m. January 20, the lower ends of the columns were more than 60 feet from the ground, and therefore far above the lamps. They extended to a height of about 300 feet, as judged by the distance of the nearest lamp and my elevation above the ground, which was about 60 feet. A narrow rift extended horizontally through the cloud, breaking each column of light into two parts. The figure is re-drawn from a rough sketch made at this time of the columns above three, only, of the numerous arc lamps.



The slight haze of minute icy particles which is common in zero weather is, I believe, called *poudrette* in Eastern Canada.

Vaughan Cornish.

Swarm of Velella.

On a former occasion while sailing in the Mediterranean I have noticed the abundance of this beautiful Hydro medusa, usually known as the "Portuguese Man-of-War," sometimes distributed over the surface of the sea, but never in numbers such as have been stranded along the northern shores during the past few days.

When taking my usual morning bathe at Mentone on April 5, I plunged into a living mass of these Siphonophora, which extended many yards from the shore. It was about the same on the 6th, when I gathered a number and preserved them in formol (5 per cent. from the 40 per cent. solution). Since then the shore has been literally strewn with them, a very disagreeable odour being emitted. At Bordighera yesterday and here to-day, nearly fifty miles from Mentone, their dried remains strew the shore and appear to be well preserved, though, of course, minus their beautiful colour. We have had no strong south winds, and I cannot see how to account for so remarkable a shoal. Isaac C. Thompson.

Alassio, April 9.

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